CBCS SCHEME

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18MAT21

Second Semester B.E. Degree Examination, June/July 2023
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, –2, 1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. **(06 Marks)**

b. If $\vec{A} = x^2y\hat{i} + y^2z\hat{j} + z^2x\hat{k}$, find
i) $\text{curl}(\text{curl}\,\vec{A})$ ii) $\text{div}(\text{curl}\,\vec{A})$ **(07 Marks)**

c. Show that $\vec{F} = \dfrac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational. **(07 Marks)**

OR

2 a. Find the workdone in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the straight line from (0, 0, 0) to (2, 1, 3). **(06 Marks)**

b. Using Green's theorem, evaluate $\oint_C (x^2 + xy)dx + (x^2 + y^2)dy$, where C is the square formed by the lines $x = \pm 1$, $y = \pm 1$. **(07 Marks)**

c. Using Gauss divergence theorem, evaluate
$\oint_S \vec{F}.\hat{n}\,ds$, where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$
over the region $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ **(07 Marks)**

Module-2

3 a. Solve $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x}$ **(06 Marks)**

b. Solve the variation of parameters methods $\dfrac{d^2y}{dx^2} + y = \sec x$ **(07 Marks)**

c. A body weighing 4.9kg is hung from a spring. A pull of 10kg will stretch the spring to 5 cm. The body is pulled down to 6cm below the static equilibrium position and then released. Find the displacement of the body from its equilibrium position at time t seconds. **(07 Marks)**

OR

4 a. Solve $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = \sin 2x$ **(06 Marks)**

b. Solve $(D^2 + 3D + 2)y = 1 + 3x + x^2$ **(07 Marks)**

c. Solve $x^2y'' - 3xy' + 4y = (1 + x)^2$ **(07 Marks)**

Module-3

5 a. Form a partial differential equation from the relation $xyz = f(x + y + z)$ **(06 Marks)**

b. Solve the Lagrange's partial differential equation
$x(y^2 - z^2)p + y(z^2 - x^2)q - z(x^2 - y^2) = 0$ **(07 Marks)**

c. With suitable assumptions derive one dimensional wave equation. **(07 Marks)**

OR

6 a. Using the method of direct integration solve

$$\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$$ **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} - 4z = 0$, given that when $x = 0$, $z = 1$ and $\frac{\partial z}{\partial x} = y$. **(07 Marks)**

c. Find all possible solutions of the one dimensional heat equation

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$ **(07 Marks)**

Module-4

7 a. Test for convergence the series

$$1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \text{........to } \infty$$ **(06 Marks)**

b. If α and β are the roots of Bessel equation $J_n(x) = 0$, prove that

$$\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = 0 \quad , \text{ for } \alpha \neq \beta.$$ **(07 Marks)**

c. Express $f(x) = 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomial. **(07 Marks)**

OR

8 a. Test for convergence the series

$$\left[\frac{2^2}{1^2} - \frac{2}{1}\right]^{-1} + \left[\frac{3^3}{2^3} - \frac{3}{2}\right]^{-2} + \left[\frac{4^4}{3^4} - \frac{4}{3}\right]^{-3} + \text{.........}\infty$$ **(06 Marks)**

b. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ **(07 Marks)**

c. Show that $P_4(\cos\theta) = \frac{1}{64}[35\cos 4\theta + 20\cos 2\theta + 9]$ **(07 Marks)**

Module-5

9 a. Using Newton's forward interpolation find y when x = 38 from the following data:

x	40	50	60	70	80	90
y	184	204	226	250	276	304

(06 Marks)

b. Using Newton - Raphson method find the root of the equation $x\sin x + \cos x = 0$ near $x = \pi$ correct to four decimal places. **(07 Marks)**

c. Using Simpson's $\frac{3}{8}^{th}$ rule, evaluate $\int_0^{0.3} \sqrt{1 - 8x^3}\, dx$ by taking seven ordinates. **(07 Marks)**

OR

10 a. Obtain Newton's divided difference interpolation polynomial and hence find f(2) from

x	3	7	9	10
f(x)	168	120	72	63

(06 Marks)

b. Find a real root of $x \log_{10} x = 1.2$ by Regula – Falsi method in three iterations, given that root lies in the interval (2, 3). **(07 Marks)**

c. Evaluate $\int_0^1 \frac{x}{1 + x^2} dx$ taking six equal sub-intervals by using Weddle's rule. **(07 Marks)**

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